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Study of Time-Fractional three Dimensional Thermoelastic Problem of a Thin Rectangular Plate

Hamna Mirza[1] and H.S. Roy[2]
[1]*Department of Mathematics,*
Swamy Vivekanand Government P.G. College, Harda, (Madhya Pradesh) India.
[2]*Department of Mathematics,*
S. B. Jain Institute of Technology, Management and Research, Nagpur 440 013, India.

(Corresponding author: Hamna Mirza)
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ABSTRACT: The present work deals with the analysis of three dimensional transient thermoelastic problem of a thin rectangular plate by application of fractional order theory occupying the region $D:-a \le x \le a\,;-b \le y \le b\,;0 \le z \le h$, with certain boundary conditions. The temperature distribution, displacement function and thermal stresses on upper plane surface of plate are determined by using double finite Marchi-Fasulo transform and Laplace transform techniques. Special case and Numerical results are discussed.

Keywords: Fractional Order Theory, thin rectangular plate, Marchi-Fasulo transform, Laplace transform, Displacement, temperature, stresses.

I. INTRODUCTION

Many models of physical processes such as heat conduction, wave propagation, diffusion, electric theories and viscoelasticity have been modified by use of Fractional calculus. In the second half of the 19th century theory of fractional integrals and derivatives was established. Abel [1] was given the first application of fractional derivatives during the formulation of the Tautochrone problem by applying fractional calculus in the solution of an integral equation. Caputo and Mainardi [2, 3] and Caputo [4] introduced model of dissipation based on memory and checked with experimental dissipation curves of various materials.

Variation of time-fractional differential operators with memory effects was investigated by Povstenko [5, 6]. Time-fractional heat conduction in a composite medium is solved analytically for an infinite matrix and is presented for a spherical inclusion by Povstenko in [7]. Associated Thermal Stresses is determined in space with a Source which Varying Harmonically in Time Space in context of Fractional Heat Conduction.

Lamba [15] determined the temperature distribution, unknown temperature gradient, displacement, stress functions and thermal stresses on the outer curved surface of a thin annular fin with known interior heat flux. Khobragade [10] discussed an inverse axially symmetric quasi-static problem of thermoelasticity for a thin clamped circular plate in which a heat flux is prescribed on an internal cylindrical surface of the plate with the help of a generalized integral transform technique. Lamba [8] studied the three-dimensional inverse transient thermoelastic problem for a thin rectangular object within the context of the theory of generalized thermoelasticity. Tikhe [16] determined the unknown heating temperature and temperature distributions on the upper surface of a thin circular plate. Lamba [9] studied the uncoupled thermoelastic response of thick cylinder of length 2h in which heat sources are generated according to the linear function of the temperature, with boundary conditions of the radiation type. Raslan [17] applied the fractional order theory of thermoelasticity to the two dimensional problem of a thick plate whose lower and upper surfaces are traction free and subjected to a given axisymmetric temperature distribution.

Here an attempt is made to determine the unknown temperature, displacement and stress function a thin rectangular plate occupying the space $D:-a \le x \le a\,; \quad -b \le y \le b\,;\ 0 \le z \le h$ subjected to certain boundary conditions. The finite Double Marchi-Fasulo transforms and Laplace transform techniques have been used to find the solution of the problem.

A. Notation and governing equations

a. The basic relationship for the problem defined above can be summarized as follows:

b. Consider a thin rectangular plate occupying the space $D:-a \le x \le a\,;\ -b \le y \le b\,;\ 0 \le z \le h$.

The displacement components u_x, u_y and u_z in the x, y and z directions respectively are in the integral as

$$u_x = \int_{-a}^{a}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-v\frac{\partial^2 U}{\partial x^2}\right)+\alpha T\right]dx \tag{1}$$

$$u_y = \int_{-b}^{b}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}-v\frac{\partial^2 U}{\partial y^2}\right)+\alpha T\right]dy \tag{2}$$

$$u_z = \int_{0}^{h}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}-v\frac{\partial^2 U}{\partial z^2}\right)+\alpha T\right]dz \tag{3}$$

Where E, v and α are the Young's modulus, Poisson's ratio and the linear coefficient of thermal expansion of the material of the plate respectively and $U(x, y, z, t)$ is the Airy's stress functions which satisfy the differential equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2 U(x, y, z, t) = -\alpha E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)T(x, y, z, t) \tag{4}$$

c. The Caputo type fractional derivative for nonlocal heat conduction is defined by [12]

$$\frac{\partial^\alpha f(t)}{\partial t^\alpha} = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1}\frac{d^n f(\tau)}{d\tau^n}d\tau, \qquad n-1<\alpha<n \tag{5}$$

To find Laplace transforms of the Caputo derivative it needs to know the initial values of the function $f(t)$ and its integer derivatives of the order $r = 0,1,2,....,n-1$

$$L\left\{\frac{\partial^\alpha f(t)}{\partial t^\alpha}\right\} = s^\alpha f^*(s) - \sum_{r=0}^{r=n-1} f^{(r)}(0^+)s^{\alpha-1-r}, \qquad n-1<\alpha<n \tag{6}$$

d. The thermal stress components in terms of the displacement components are given as,

The stress components in terms of $U(x, y, z, t)$ are given by

$$\sigma_{xx} = \left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right) \tag{7}$$

$$\sigma_{yy} = \left(\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}\right) \tag{8}$$

$$\sigma_{zz} = \left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right) \tag{9}$$

B. Formulation of the Problem

Consider a thin rectangular plate occupying the space $D: -a \le x \le a;\ \ -b \le y \le b;\ \ 0 \le z \le h$ in the Application of Fractional Order Theory.

The heat conduction equation in time fractional order context for thin rectangular plate is given as

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial^\alpha T}{\partial t^\alpha} \tag{10}$$

where k is the thermal diffusivity of the material.

Subject to the initial condition

$$T(x, y, z, 0) = 0 \tag{11}$$

the boundary conditions

$$\left[T(x, y, z, t) + k_1 \frac{\partial T(x, y, z, t)}{\partial x}\right]_{x=-a} = G_1(y, z, t) \tag{12}$$

$$\left[T(x, y, z, t) + k_2 \frac{\partial T(x, y, z, t)}{\partial x}\right]_{x=a} = G_2(y, z, t) \tag{13}$$

$$\left[T(x, y, z, t) + k_3 \frac{\partial T(x, y, z, t)}{\partial y}\right]_{y=b} = F_1(x, z, t) \tag{14}$$

$$\left[T(x, y, z, t) + k_4 \frac{\partial T(x, y, z, t)}{\partial y}\right]_{y=-b} = F_2(x, z, t) \tag{15}$$

$$\left[T(x, y, z, t) + c \frac{\partial T(x, y, z, t)}{\partial z}\right]_{z=0} = g(x, y, t) \tag{16}$$

$$\left[\overline{T}(x, y, z, t)\right]_{z=h} = \overline{\psi}(x, y, t) \tag{17}$$

The equations (1) to (17) constitute the mathematical formulation of the problem under consideration.

C. Solution of the Problem

To obtain the expression for the temperature function $T(x, y, z, t)$, we develop the finite Marchi-Fasulo integral transform as

$$\overline{F}(n) = \int_{-h}^{h} f(z) P_n(z)\, dz \tag{18}$$

then at each point of (*-h, h*) at which *f(z)* is continuous,

$$f(z) = \sum_{n=1}^{\infty} \frac{\overline{F}(n)}{\lambda_n} P_n(z) \tag{19}$$

where $P_n(z) = Q_n \cos(a_n z) - W_n \sin(a_n z)$

$$Q_n = a_n(\alpha_1 + \alpha_2)\cos(a_n h) + (\beta_1 - \beta_2)\sin(a_n h)$$

$$W_n = (\beta_1 + \beta_2)\cos(a_n h) + (\alpha_2 - \alpha_1) a_n \sin(a_n h)$$

$$\lambda_n = \int_{-h}^{h} P_n^2(z)\, dz = h\left[Q_n^2 + W_n^2\right] + \frac{\sin(2a_n h)}{2a_n}\left[Q_n^2 - W_n^2\right]$$

The eigen values a_n are the solutions of the equation

$$\left[\alpha_1 a \cos(ah) + \beta_1 \sin(ah)\right] \times \left[\beta_2 \cos(ah) + \alpha_2 a \sin(ah)\right]$$
$$= \left[\alpha_2 a \cos(ah) - \beta_2 \sin(ah)\right] \times \left[\beta_1 \cos(ah) - \alpha_1 a \sin(ah)\right] \tag{20}$$

α_1, α_2, β_1 and β_2 are constants.

The sum in (19) must be taken on n corresponding to the positive roots of the equation (18).

Moreover the integral transform (18) has the following property:

$$\int_{-h}^{h} \frac{\partial^2 f(z)}{\partial z^2} P_n(z)\, dz = \frac{P_n(h)}{\alpha_1}\left[\beta_1 f(z) + \alpha_1 \frac{\partial f(z)}{\partial z}\right]_{z=h} - \frac{P_n(-h)}{\alpha_2}\left[\beta_2 f(z) + \alpha_2 \frac{\partial f(z)}{\partial z}\right]_{z=-h} - a_n^2 \overline{F}(n)$$

Applying finite **Marchi-Fasulo integral transform** defined in equations (18) to (20) to the equations (10) and using the boundary conditions (11) to (17), one obtains

$$\frac{d^2\overline{T}}{dy^2}+\frac{d^2\overline{T}}{dz^2}-a_m{}^2\overline{T}=\frac{1}{k}\frac{d^\alpha\overline{T}}{dt^\alpha}+G(n,z,t) \tag{21}$$

Where the Eigen values a_n are the solution of the equation

$$\left[\alpha_1 a\cos(a^2)+\beta_1\sin(a^2)\right]\times\left[\beta_2\cos(a^2)+\alpha_2 a\sin(a^2)\right]$$
$$=\left[\alpha_2 a\cos(a^2)-\beta_2\sin(a^2)\right]\times\left[\beta_1\cos(a^2)-\alpha_1 a\sin(a^2)\right]$$

$$\overline{T}(m,y,z,0)=0 \tag{22}$$

$$\left[\overline{T}(m,y,z,t)+k_3\frac{d\overline{T}(m,y,z,t)}{dy}\right]_{y=b}=\overline{F_1}(m,z,t) \tag{23}$$

$$\left[\overline{T}(m,y,z,t)+k_4\frac{d\overline{T}(m,y,z,t)}{dy}\right]_{y=-b}=\overline{F_2}(m,z,t) \tag{24}$$

$$\left[\overline{T}(m,y,z,t)+c\frac{d\overline{T}(m,y,z,t)}{dz}\right]_{z=0}=\overline{g}(m,y,t) \tag{25}$$

$$\left[\overline{T}(m,y,z,t)\right]_{z=h}=\overline{\psi}(m,y,t) \tag{26}$$

Where $\overline{T}$ denotes the Marchi-Fasulo transform of T and m is the Marchi-Fasulo transform parameter.
Again applying finite **Marchi-Fasulo integral transform** to the equations (21) and using (22) to (26) one obtains

$$\frac{d^2\overline{\overline{T}}}{dz^2}-q^2\overline{\overline{T}}=\frac{1}{k}\frac{d^\alpha\overline{\overline{T}}}{dt^\alpha}+H(m,n,z,t) \tag{27}$$

where $a_p{}^2=a_n{}^2+b_m{}^2$,

in which the eigen values b_m are the solutions of the equation

$$\left[\alpha_3 a\cos(ab)+\beta_4\sin(ab)\right]\times\left[\beta_4\cos(ab)+\alpha_4 a\sin(ab)\right]$$
$$=\left[\alpha_4 a\cos(ab)-\beta_4\sin(ab)\right]\times\left[\beta_3\cos(ab)-\alpha_3 a\sin(ab)\right]$$

$$\overline{\overline{T}}(m,n,z,0)=0 \tag{28}$$

$$\left[\overline{\overline{T}}(m,n,z,t)+c\frac{d\overline{\overline{T}}(m,n,z,t)}{dz}\right]_{z=0}=\overline{\overline{g}}(m,n,t) \tag{29}$$

$$\left[\overline{T}(m,y,z,t)\right]_{z=h}=\overline{\psi}(m,y,t) \tag{30}$$

where $\overline{\overline{T}}$ denotes the Marchi-Fasulo transform of $\overline{T}$ and n is the Marchi-Fasulo transform parameter, α_1, α_2, α_3, α_4, and β_1 , β_2, β_3, β_4 are constants.

Further applying **Laplace transform** stated in (6) to the equations (27) and using (28) to (30) one obtains

$$\frac{d^2\overline{\overline{T}}^*}{dz^2} - q^2\overline{\overline{T}}^* = H^*(m,n,z,s) \tag{31}$$

where $q^2 = a_p{}^2 + \frac{1}{k}\left[s^\alpha L\{\theta\} - \sum_{r=0}^{r=n-1} \theta^{(r)}(0^+) s^{\alpha-1-r} \right]$

$$\left[\overline{\overline{T}}^*(m,n,z,s) + c\frac{d\overline{\overline{T}}^*(m,n,z,s)}{dz} \right]_{z=0} = \overline{\overline{g}}^*(m,n,s) \tag{32}$$

$$\left[\overline{\overline{T}}^*(m,y,z,s) \right]_{z=h} = \overline{\overline{\psi}}^*(m,y,s) \tag{33}$$

where $\overline{\overline{T}}^*$ denotes the Laplace transform of $\overline{\overline{T}}$ and s is a Laplace transform parameter.
The equation (31) is a **second order differential equation** whose solution is given by

$$\overline{\overline{T}}^*(m,n,z,s) = Ae^{qz} + Be^{-qz} + P.I \tag{34}$$

where A, B are arbitrary constants and P.I stands for particular integral and is given by

$$P.I = \frac{H^*(m,n,z,s)}{D^2 - q^2}$$

Using (32) and (33) in (34) we obtain the values of A and B. Substituting these values in (34) and then **inversion** of **Laplace transform** and finite **Marchi-Fasulo transform**, one obtains

$$\begin{aligned} T(x,y,z,t) &= 2k\pi \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n(y)}{\lambda_n}\right) \frac{m(-1)^m}{\xi^2 + c^2m^2\pi^2} \left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi z}{\xi}\right) - \sin\left(\frac{m\pi z}{\xi}\right) \right] \\ &\times \int_0^t \left[f(m,n,t') - \left[P.I + c\frac{d}{dz}(P.I) \right]_{z=\xi} \right] E_\alpha\left(-k\left(a_p{}^2 + \frac{m^2\pi^2}{\xi^2} \right)(t^\alpha - t'^\alpha) \right) \\ &- 2k\pi \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n(y)}{\lambda_n}\right) \frac{m(-1)^m}{\xi^2 + c^2m^2\pi^2} \left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi(z-\xi)}{\xi}\right) - \sin\left(\frac{m\pi(z-\xi)}{\xi}\right) \right] \\ &\times \int_0^t \left[g(m,n,t') - \left[P.I + c\frac{d}{dz}(P.I) \right]_{z=0} \right] E_\alpha\left(-k\left(a_p{}^2 + \frac{m^2\pi^2}{\xi^2} \right)(t^\alpha - t'^\alpha) \right) \\ &+ \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n(y)}{\lambda_n}\right) L^{-1}(P.I) \end{aligned} \tag{35}$$

Here $E_\alpha(.)$ represents Mittag-Leffler function.

$$\begin{aligned} \psi(x,y,t) &= 2k\pi \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n(y)}{\lambda_n}\right) \frac{m(-1)^m}{\xi^2 + c^2m^2\pi^2} \left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi h}{\xi}\right) - \sin\left(\frac{m\pi h}{\xi}\right) \right] \\ &\times \int_0^t \left[f(m,n,t') - \left[P.I + c\frac{d}{dz}(P.I) \right]_{z=\xi} \right] E_\alpha\left(-k\left(a_p{}^2 + \frac{m^2\pi^2}{\xi^2} \right)(t^\alpha - t'^\alpha) \right) \end{aligned}$$

$$-2k\pi\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n(y)}{\lambda_n}\right)\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi(h-\xi)}{\xi}\right)-\sin\left(\frac{m\pi(h-\xi)}{\xi}\right)\right]$$

$$\times\int_0^t\left[g(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=0}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n(y)}{\lambda_n}\right)L^{-1}(P.I) \qquad (36)$$

in which $\bar{\bar{f}}(m,n,t)$ and $\bar{\bar{g}}(m,n,t)$ denote the Marchi-Fasulo transform of $\bar{f}(m,y,t)$ and $\bar{g}(m,y,t)$ respectively, $\bar{f}(m,y,t)$ and $\bar{g}(m,y,t)$ denote the finite Marchi-Fasulo transform of $f(x,y,z,t)$ and $g(x,y,z,t)$ respectively,

$$\bar{\bar{f}}(m,n,t)=\int_{-b}^{b}\bar{f}(m,y,t)P_n(y)\,dy\ ,\quad \bar{\bar{g}}(m,n,t)=\int_{-b}^{b}\bar{g}(m,y,t)P_n(y)\,dy\ ,\quad \lambda_n=\int_{-b}^{b}P_n{}^2(y)\,dy\ ,$$

$$P_m(x)=Q_m\cos(a_mx)-W_m\sin(a_mx)$$

$$Q_m=a_m(\alpha_1+\alpha_2)\cos(a_ma)+(\beta_1-\beta_2)\sin(a_ma)\ ,$$

$$W_m=(\beta_1+\beta_2)\cos(a_ma)+(\alpha_2-\alpha_1)a_m\sin(a_ma)$$

$$P_n(y)=Q_n\cos(b_ny)-W_n\sin(b_ny)\ ,$$

$$Q_n=b_n(\alpha_1+\alpha_2)\cos(a_nb)+(\beta_1-\beta_2)\sin(b_nb)\ ,$$

$$W_n=(\beta_1+\beta_2)\cos(b_nb)+(\alpha_2-\alpha_1)b_n\sin(b_nb)\ ,$$

Equation (35) and (36) are the desired solution of the given problem with $\beta_1=\beta_2=\beta_3=\beta_4=1$ and $\alpha_1=k_1$, $\alpha_2=k_2$, $\alpha_3=k_3$, $\alpha_4=k_4$.

D. Airy's Stress Function

Substituting the value of temperature distribution $T(x,y,z,t)$ from equation (35) in equation (4) one obtains the expression for Airy's stress function $U(x,y,z,t)$ as

$$U(x,y,z,t)=-2k\pi\alpha E\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n(y)}{\lambda_n}\right)\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi z}{\xi}\right)-\sin\left(\frac{m\pi z}{\xi}\right)\right]$$

$$\times\int_0^t\left[f(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=\xi}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$+2k\pi\alpha E\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n(y)}{\lambda_n}\right)\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi(z-\xi)}{\xi}\right)-\sin\left(\frac{m\pi(z-\xi)}{\xi}\right)\right]$$

$$\times\int_0^t\left[g(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=0}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-\alpha E\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n(x)}{\lambda_n}\right)L^{-1}(P.I) \qquad (37)$$

E. Displacement Components

Substituting the value of Airy's stress function $U(x, y, z, t)$ from (37) in the equations (1) to (3) one obtains

$$u_x(x,y,z,t) = -2k\pi\alpha\left[\sum_{n=1}^{\infty}\left(\frac{P_n''(y)}{\lambda_n}\right)\right]\sum_{m=1}^{\infty}\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi c}{\xi}\right)-\sin\left(\frac{m\pi c}{\xi}\right)\right]\int_{-a}^{a}\frac{P_m(x)}{\mu_m}dx$$

$$\times\int_0^t\left[f(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=\xi}\right]E_\alpha\left(-k\left(a_p^{\,2}+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$+2k\pi\alpha\left[\sum_{n=1}^{\infty}\left(\frac{P_n(y)}{\lambda_n}\right)\right]\sum_{m=1}^{\infty}\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left(\frac{m\pi}{\xi}\right)\left[\frac{m\pi c}{\xi}\sin\left(\frac{m\pi z}{\xi}\right)+\cos\left(\frac{m\pi z}{\xi}\right)\right]\int_{-a}^{a}\frac{P_m(x)}{\mu_m}dx$$

$$\times\int_0^t\left[f(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=\xi}\right]E_\alpha\left(-k\left(a_p^{\,2}+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$+2k\pi\alpha\left[\sum_{n=1}^{\infty}\left(\frac{P_n(y)}{\lambda_n}\right)\right]\sum_{m=1}^{\infty}\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi z}{\xi}\right)-\sin\left(\frac{m\pi z}{\xi}\right)\right]\int_{-a}^{a}\frac{P_m(x)+\nu P_m''(x)}{\mu_m}dx$$

$$\times\int_0^t\left[f(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=\xi}\right]E_\alpha\left(-k\left(a_p^{\,2}+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$+2k\pi\alpha\left[\sum_{n=1}^{\infty}\left(\frac{P_n''(y)}{\lambda_n}\right)\right]\sum_{m=1}^{\infty}\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi(z-\xi)}{\xi}\right)-\sin\left(\frac{m\pi(z-\xi)}{\xi}\right)\right]\int_{-a}^{a}\frac{P_m(x)}{\mu_m}dx$$

$$\times\int_0^t\left[g(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=0}\right]E_\alpha\left(-k\left(a_p^{\,2}+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-2k\pi\alpha\left[\sum_{n=1}^{\infty}\left(\frac{P_n(y)}{\lambda_n}\right)\right]\sum_{m=1}^{\infty}\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left(\frac{m\pi}{\xi}\right)\left[\frac{m\pi c}{\xi}\sin\left(\frac{m\pi(z-\xi)}{\xi}\right)+\cos\left(\frac{m\pi(z-\xi)}{\xi}\right)\right]\int_{-a}^{a}\frac{P_m(x)}{\mu_m}dx$$

$$\times\int_0^t\left[g(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=0}\right]E_\alpha\left(-k\left(a_p^{\,2}+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-2k\pi\alpha\left[\sum_{n=1}^{\infty}\left(\frac{P_n(y)}{\lambda_n}\right)\right]\sum_{m=1}^{\infty}\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi(z-\xi)}{\xi}\right)-\sin\left(\frac{m\pi(z-\xi)}{\xi}\right)\right]\int_{-a}^{a}\frac{P_m(x)+\nu P_m''(x)}{\mu_m}dx$$

$$\times\int_0^t\left[g(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=0}\right]E_\alpha\left(-k\left(a_p^{\,2}+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-\alpha\left[\sum_{n=1}^{\infty}\frac{P_n(y)+P_n''(y)}{\lambda_n}\right]\sum_{m=1}^{\infty}\int_{-a}^{a}\frac{P_m(x)}{\mu_m}dx\;L^{-1}(P.I)$$

$$+\alpha\left[\sum_{n=1}^{\infty}\frac{P_n(y)}{\lambda_n}\right]\sum_{m=1}^{\infty}\int_{-a}^{a}\frac{P_n(x)+\nu P_m''(x)}{\mu_m}dx\;L^{-1}(P.I) \tag{38}$$

$$u_y(x,y,z,t)=2k\pi\alpha\left[\sum_{m=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\right]\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left(\frac{m\pi}{\xi}\right)\left[\frac{m\pi c}{\xi}\sin\left(\frac{m\pi z}{\xi}\right)+\cos\left(\frac{m\pi z}{\xi}\right)\right]\int_{-b}^{b}\frac{P_n(y)}{\lambda_n}dy$$

$$\times\int_0^t\left[f(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=\xi}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-2k\pi\alpha\sum_{m=1}^{\infty}\left(\frac{P_m''(x)}{\mu_m}\right)\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi z}{\xi}\right)-\sin\left(\frac{m\pi z}{\xi}\right)\right]\int_{-b}^{b}\frac{P_n(y)}{\lambda_n}dy$$

$$\times\int_0^t\left[f(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=\xi}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$+2k\pi\alpha\left[\sum_{m=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\right]\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi z}{\xi}\right)-\sin\left(\frac{m\pi z}{\xi}\right)\right]\int_{-b}^{b}\frac{P_n(y)+\nu P_n''(y)}{\lambda_n}dy$$

$$\times\int_0^t\left[f(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=\xi}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-2k\pi\alpha\left[\sum_{m=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\right]\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left(\frac{m\pi}{\xi}\right)\left[\frac{m\pi c}{\xi}\sin\left(\frac{m\pi(z-\xi)}{\xi}\right)+\cos\left(\frac{m\pi(z-\xi)}{\xi}\right)\right]\int_{-b}^{b}\frac{P_n(y)}{\lambda_n}dy$$

$$\times\int_0^t\left[g(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=0}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$+2k\pi\alpha\left[\sum_{m=1}^{\infty}\left(\frac{P_m''(x)}{\mu_m}\right)\right]\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi(z-\xi)}{\xi}\right)-\sin\left(\frac{m\pi(z-\xi)}{\xi}\right)\right]\int_{-b}^{b}\frac{P_n(y)}{\lambda_n}dy$$

$$\times\int_0^t\left[g(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=0}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-2k\pi\alpha\left[\sum_{m=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\right]\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi(z-\xi)}{\xi}\right)-\sin\left(\frac{m\pi(z-\xi)}{\xi}\right)\right]\int_{-b}^{b}\frac{P_n(y)+\nu P_n''(y)}{\lambda_n}dy$$

$$\times\int_0^t\left[g(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=0}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-\alpha\left[\sum_{m=1}^{\infty}\frac{P_m(x)+P_m''(x)}{\mu_m}\right]\int_{-a}^{a}\frac{P_n(y)}{\lambda_n}dyL^{-1}(P.I)$$

$$+\alpha\left[\sum_{m=1}^{\infty}\frac{P_m(x)}{\mu_m}\right]\int_{-a}^{a}\frac{P_n(y)+\nu P_n''(y)}{\lambda_n}dy\ L^{-1}(P.I) \tag{39}$$

$$u_z(x,y,z,t)=-\left[2k\pi\alpha\left[\sum_{m=1}^{\infty}\frac{P_m''(x)}{\mu_m}\sum_{n=1}^{\infty}\frac{P_n(y)}{\lambda_n}+\sum_{m=1}^{\infty}\frac{P_m(x)}{\mu_m}\sum_{n=1}^{\infty}\frac{P_n''(y)}{\lambda_n}\right]\right]$$

$$\sum_{m=1}^{\infty} \frac{m(-1)^m}{\xi^2 + c^2 m^2 \pi^2}\left[c \sin\left(\frac{m\pi h}{\xi}\right) + \frac{\xi}{m\pi}\left[\cos\left(\frac{m\pi h}{\xi}\right) - 1\right]\right]$$

$$\times \int_0^t \left[f(m,n,t') - \left[P.I + c\frac{d}{dz}(P.I)\right]_{z=\xi}\right] E_\alpha\left(-k\left(a_p{}^2 + \frac{m^2\pi^2}{\xi^2}\right)(t^\alpha - t'^\alpha)\right)$$

$$+ 2k\pi\alpha \sum_{m=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\sum_{n=1}^{\infty}\left(\frac{P_n(y)}{\lambda_n}\right)\sum_{m=1}^{\infty} \frac{m(-1)^m}{\xi^2 + c^2 m^2 \pi^2}\left[c \sin\left(\frac{m\pi h}{\xi}\right) + \frac{\xi}{m\pi}\left[\cos\left(\frac{m\pi h}{\xi}\right) - 1\right]\right]$$

$$- \nu\left[\sin\left(\frac{m\pi h}{\xi}\right) - \frac{mc\pi}{\xi}\left[\cos\left(\frac{m\pi h}{\xi}\right) - 1\right]\right]$$

$$\times \int_0^t \left[f(m,n,t') - \left[P.I + c\frac{d}{dz}(P.I)\right]_{z=\xi}\right] E_\alpha\left(-k\left(a_p{}^2 + \frac{m^2\pi^2}{\xi^2}\right)(t^\alpha - t'^\alpha)\right)$$

$$+ 2k\pi\alpha\left[\sum_{m=1}^{\infty}\left(\frac{P_m''(x)}{\mu_m}\right)\sum_{n=1}^{\infty}\frac{P_n(y)}{\lambda_n} + \sum_{m=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\sum_{n=1}^{\infty}\frac{P_n''(y)}{\lambda_n}\right]$$

$$\sum_{m=1}^{\infty} \frac{m(-1)^m}{\xi^2 + c^2 m^2 \pi^2}\left[c \sin\left(\frac{m\pi(h-\xi)}{\xi}\right) + \frac{\xi}{m\pi}\left[\cos\left(\frac{m\pi(h-\xi)}{\xi}\right) - (-1)^m\right]\right]$$

$$\times \int_0^t \left[g(m,n,t') - \left[P.I + c\frac{d}{dz}(P.I)\right]_{z=0}\right] E_\alpha\left(-k\left(a_p{}^2 + \frac{m^2\pi^2}{\xi^2}\right)(t^\alpha - t'^\alpha)\right)$$

$$+ 2k\pi\alpha\left[\sum_{m=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\sum_{n=1}^{\infty}\left(\frac{P_n(y)}{\lambda_n}\right)\right]\sum_{m=1}^{\infty}\frac{m(-1)^m}{\xi^2 + c^2 m^2\pi^2}\left[\nu\left[\sin\left(\frac{m\pi(h-\xi)}{\xi}\right) - \left(\frac{m\pi c}{\xi}\right)\left[\cos\left(\frac{m\pi(h-\xi)}{\xi}\right) - (1)^m\right]\right]\right]$$

$$- \left[c \sin\left(\frac{m\pi(h-\xi)}{\xi}\right) + \left(\frac{\xi}{m\pi}\right)\left[\cos\left(\frac{m\pi(h-\xi)}{\xi}\right) - (-1)^m\right]\right]$$

$$\times \int_0^t \left[g(m,n,t') - \left[P.I + c\frac{d}{dz}(P.I)\right]_{z=0}\right] E_\alpha\left(-k\left(a_p{}^2 + \frac{m^2\pi^2}{\xi^2}\right)(t^\alpha - t'^\alpha)\right)$$

$$- h\alpha\left[\sum_{m=1}^{\infty}\left(\frac{P_m''(x)}{\mu_m}\right)\sum_{n=1}^{\infty}\frac{P_n(y)}{\lambda_n} + \sum_{m=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\sum_{n=1}^{\infty}\frac{P_n''(y)}{\lambda_n}\right] L^{-1}[P.I.]$$

$$+ (\nu+1)\alpha h \sum_{m=1}^{\infty}\frac{P_m(x)}{\mu_m}\sum_{n=1}^{\infty}\frac{P_n(y)}{\lambda_n} \quad L^{-1}(P.I) \tag{40}$$

F. Stress Functions

Substituting the value of $U(x, y, z, t)$ from equation (37) in the equations (7) to (9), one obtain

$$\sigma_{xx}(x,y,z,t) = -2k\pi\alpha E\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n''(y)}{\lambda_n}\right)\frac{m(-1)^m}{\xi^2 + c^2 m^2\pi^2}\right]\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi z}{\xi}\right) - \sin\left(\frac{m\pi z}{\xi}\right)\right]$$

$$\times \int_0^t \left[f(m,n,t') - \left[P.I + c\frac{d}{dz}(P.I)\right]_{z=\xi}\right] E_\alpha\left(-k\left(a_p{}^2 + \frac{m^2\pi^2}{\xi^2}\right)(t^\alpha - t'^\alpha)\right)$$

$$+2k\pi\alpha E\left[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n(y)}{\lambda_n}\right)\left(\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\right)\right]\left(\frac{m\pi}{\xi}\right)\left[\frac{m\pi c}{\xi}\sin\left(\frac{m\pi z}{\xi}\right)+\cos\left(\frac{m\pi z}{\xi}\right)\right]$$

$$\times\int_0^t\left[f(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=\xi}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$+2k\pi\alpha E\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n''(y)}{\lambda_n}\right)\left(\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\right)\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi(z-\xi)}{\xi}\right)-\sin\left(\frac{m\pi(z-\xi)}{\xi}\right)\right]$$

$$\times\int_0^t\left[g(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=0}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-2k\pi\alpha E\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n(y)}{\lambda_n}\right)\left(\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\right)\left(\frac{m\pi}{\xi}\right)\left[\frac{m\pi c}{\xi}\sin\left(\frac{m\pi(z-\xi)}{\xi}\right)+\cos\left(\frac{m\pi(z-\xi)}{\xi}\right)\right]$$

$$\times\int_0^t\left[g(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=0}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-\alpha E\left[\sum_{m=1}^{\infty}\frac{P_m(x)}{\mu_m}\right]\left[\sum_{n=1}^{\infty}\frac{P_n(y)+P_n''(y)}{\lambda_n}\right]L^{-1}(P.I) \tag{41}$$

$$\sigma_{yy}(x,y,z,t)=2k\pi\alpha E\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n(x)}{\lambda_n}\right)\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\right]\left(\frac{m\pi}{\xi}\right)\left[\frac{m\pi c}{\xi}\sin\left(\frac{m\pi z}{\xi}\right)+\cos\left(\frac{m\pi z}{\xi}\right)\right]$$

$$\times\int_0^t\left[f(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=\xi}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-2k\pi\alpha E\left[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\frac{P_m''(x)}{\mu_m}\right)\left(\frac{P_n(y)}{\lambda_n}\right)\left(\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\right)\right]\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi z}{\xi}\right)-\sin\left(\frac{m\pi z}{\xi}\right)\right]$$

$$\times\int_0^t\left[f(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=\xi}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-2k\pi\alpha E\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n(y)}{\lambda_n}\right)\left(\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\right)\left(\frac{m\pi}{\xi}\right)\left[\frac{m\pi c}{\xi}\sin\left(\frac{m\pi(z-\xi)}{\xi}\right)+\cos\left(\frac{m\pi(z-\xi)}{\xi}\right)\right]$$

$$\times\int_0^t\left[g(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=0}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$2k\pi\alpha E\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_m''(x)}{\mu_m}\right)\left(\frac{P_n(y)}{\lambda_n}\right)\left(\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\right)\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi(z-\xi)}{\xi}\right)-\sin\left(\frac{m\pi(z-\xi)}{\xi}\right)\right]$$

$$\times\int_0^t\left[g(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=0}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-\alpha E\left[\sum_{n=1}^{\infty}\frac{P_n(y)}{\lambda_n}\right]\left[\sum_{m=1}^{\infty}\frac{P_m(x)+P_m''(x)}{\mu_m}\right] L^{-1}(P.I) \tag{42}$$

$$\sigma_{zz}(x,y,z,t)=-2k\pi\alpha E\left[\sum_{m=1}^{\infty}\left(\frac{P_m''(x)}{\mu_m}\right)\sum_{n=1}^{\infty}\left(\frac{P_n(y)}{\lambda_n}\right)+\sum_{m=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\sum_{n=1}^{\infty}\left(\frac{P_n''(y)}{\lambda_n}\right)\right]$$

$$\left[\sum_{m=1}^{\infty}\left(\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\right)\right]\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi z}{\xi}\right)-\sin\left(\frac{m\pi z}{\xi}\right)\right]$$

$$\times\int_0^t\left[f(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=\xi}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$+2k\pi\alpha E\left[\sum_{m=1}^{\infty}\left(\frac{P_m''(x)}{\mu_m}\right)\sum_{n=1}^{\infty}\left(\frac{P_n(y)}{\lambda_n}\right)+\sum_{m=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\sum_{n=1}^{\infty}\left(\frac{P_n''(y)}{\lambda_n}\right)\right]$$

$$\sum_{m=1}^{\infty}\frac{m(-1)^m}{\xi^2+c^2m^2\pi^2}\left[\frac{m\pi c}{\xi}\cos\left(\frac{m\pi(z-\xi)}{\xi}\right)-\sin\left(\frac{m\pi(z-\xi)}{\xi}\right)\right]$$

$$\times\int_0^t\left[g(m,n,t')-\left[P.I+c\frac{d}{dz}(P.I)\right]_{z=0}\right]E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-\alpha E\left[\sum_{m=1}^{\infty}\left(\frac{P_m''(x)}{\mu_m}\right)\sum_{n=1}^{\infty}\left(\frac{P_n(y)}{\lambda_n}\right)+\sum_{m=1}^{\infty}\left(\frac{P_m(x)}{\mu_m}\right)\sum_{n=1}^{\infty}\left(\frac{P_n''(y)}{\lambda_n}\right)\right].L^{-1}(P.I) \tag{43}$$

G. Special Case and Numerical Results

Set $f(x,y,t)=(1-e^{-t})(x+a)^2(x-a)^2(y+b)^2(y-b)^2$,

$g(x,y,t)=(1-e^{-t})(x+a)^2(x-a)^2(y+b)^2(y-b)^2e^h$, $\delta=\dfrac{8(k_1+k_2)k\pi}{h^2}$, $a=0.75, k=1,$

$b=2,\ h=1,\ t=1$ in the equation (35) to obtain

$$\frac{T(x,y,z,t)}{\delta}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{\eta=1}^{\infty}(-1)^{(\eta+1/2)}\left(\eta+\frac{1}{2}\right)\left(\frac{P_m(x)}{\mu_m}\right)\left(\frac{P_n(y)}{\lambda_n}\right)\left(\frac{1}{1-a_p{}^2}\right)$$

$$\times\left\{\left[\frac{a_n\cos^2(a_n)-\cos(a_n)\sin(a_n)}{a_n{}^2}\right]-\left[\frac{b_m\cos^2(b_m)-\cos(b_m)\sin(b_m)}{b_m{}^2}\right]e\right\}$$

$$\times E_\alpha\left(-k\left(a_p{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

CONCLUSIONS

In this work, the temperature, displacement and thermal stresses has been determined for thin rectangular plate in context of fractional order theory of thermoelasticity. The Double finite Marchi-Fasulo and Laplace transform techniques have been used to solve the problem. The system of equations proposed in this study can be adapted to design of useful structures or machines in engineering applications.

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